**Exercise 6:**

**Greedy Algorithm:**

**Greedy is an algorithmic paradigm that builds up a solution piece by piece, always choosing the next piece that offers the most obvious and immediate benefit. Greedy algorithms are used for optimization problems. *At every step, we can make a choice that looks best at the moment, and we get the optimal solution of the complete problem*.**

**Exercise:**

1. [Kruskal’s Minimum Spanning Tree Algorithm](http://www.geeksforgeeks.org/greedy-algorithms-set-2-kruskals-minimum-spanning-tree-mst/)
2. [Prim’s Minimum Spanning Tree Algorithm](http://www.geeksforgeeks.org/greedy-algorithms-set-5-prims-minimum-spanning-tree-mst-2/)
3. [Dijkstra’s Shortest Path Algorithm](http://www.geeksforgeeks.org/greedy-algorithms-set-6-dijkstras-shortest-path-algorithm/)
4. [Huffman Coding](http://www.geeksforgeeks.org/greedy-algorithms-set-3-huffman-coding/)
5. [Activity Selection Problem](http://www.geeksforgeeks.org/greedy-algorithms-set-1-activity-selection-problem/)
6. [Job Sequencing Problem](http://www.geeksforgeeks.org/job-sequencing-problem-set-1-greedy-algorithm/)
7. [Greedy Algorithm to find Minimum number of Coins](http://geeksquiz.com/greedy-algorithm-to-find-minimum-number-of-coins/)
8. [Minimum Number of Platforms Required for a Railway/Bus Station](http://www.geeksforgeeks.org/minimum-number-platforms-required-railwaybus-station/)
9. [Graph coloring](http://www.geeksforgeeks.org/graph-coloring-set-2-greedy-algorithm/)
10. [Fractional Knapsack Problem](http://www.geeksforgeeks.org/fractional-knapsack-problem/)
11. [Minimize Cash Flow among a given set of friends who have borrowed money from each other](http://www.geeksforgeeks.org/minimize-cash-flow-among-given-set-friends-borrowed-money/)
12. [Find minimum time to finish all jobs with given constraints](http://www.geeksforgeeks.org/find-minimum-time-to-finish-all-jobs-with-given-constraints/)
13. **Implementation of Krushkal’s Algorithm**

**Algorithm:**

* Kruskal’s algorithm also finds the minimum cost spanning tree of a graph by adding edges one-by-one.

enqueue edges of G in a queue in increasing order of cost.

T = φ ;

while(queue is not empty){

dequeue an edge e;

if(e does not create a cycle with edges in T)

add e to T;

}

return T;

1. **Implementation of Prims Algorithm:**

**Algorithm:**

1. Prim’s algorithm finds a minimum cost spanning tree by selecting edges from the graph one-by-one as follows:
2. It starts with a tree, T, consisting of the starting vertex, x.
3. Then, it adds the shortest edge emanating from x that connects T to the rest of the graph.
4. It then moves to the added vertex and repeats the process.

Consider a graph G=(V, E);

Let T be a tree consisting of only the starting vertex **x;**

while (T has fewer than IVI vertices)

{

find a smallest edge connecting T to G-T;

add it to T;

}

1. **Implementation of Dijkstra’s Shortest Path Algorithm**

**Algorithm:**

**Algorithm**:

1. Assign to every node a distance value. Set it to zero for our initial node and to infinity for

all other nodes.

2. Mark all nodes as unvisited. Set initial node as current.

3. For current node, consider all its unvisited neighbors and calculate their tentative distance

(from the initial node). For example, if current node (A) has distance of 6, and an edge

connecting it with another node (B) is 2, the distance to B through A will be 6+2=8. If

this distance is less than the previously recorded distance (infinity in the beginning, zero

for the initial node), overwrite the distance.

4. When all neighbors are considered for the current node, mark it as visited. A

visited node will not be checked ever again; its distance recorded now is final and

minimal.

5. If all nodes have been visited, finish. Otherwise, set the unvisited node with the smallest

distance (from the initial node) as the next "current node" and continue from step 3.

1. **Implementation of Huffman Coding**

**Algorithm:**

1. Scan text to be compressed and tally occurrence of all characters.

2. Sort or prioritize characters based on number of occurrences in text.

3. Build Huffman code tree based on prioritized list.

4. Perform a traversal of tree to determine all code words.

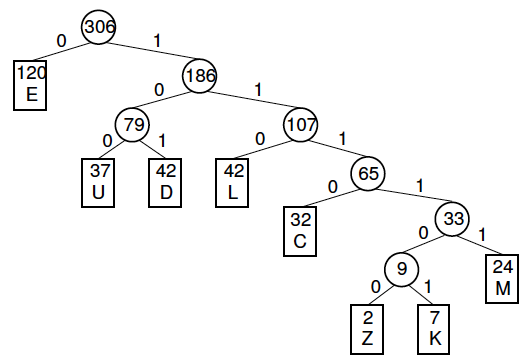
5. Scan text again and create new file using the Huffman codes.

Letter freqency table

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Letter | Z | K | M | C | U | D | L | E |
| Frequency | 2 | 7 | 24 | 32 | 37 | 42 | 42 | 120 |

Huffman code

|  |  |  |  |
| --- | --- | --- | --- |
| Letter | Freq | Code | Bits |
|  |  |  |  |
| E | 120 | 0 | 1 |
| D | 42 | 101 | 3 |
| L | 42 | 110 | 3 |
| U | 37 | 100 | 3 |
| C | 32 | 1110 | 4 |
| M | 24 | 11111 | 5 |
| K | 7 | 111101 | 6 |
| Z | 2 | 111100 | 6 |

The Huffman tree (Shaffer Fig. 5.24) 

Three problems:

* Problem 1: Huffman tree building
* Problem 2: Encoding
* Problem 3: Decoding

**Problem 2: Encoding**

Encoding a string can be done by replacing each letter in the string with its binary code (the Huffman code).

Examples:   
DEED 10100101 (8 bits)   
MUCK 111111001110111101 (18 bits)

**Problem 3: Decoding**

Decoding an encoded string can be done by looking at the bits in the coded string from left to right until a letter decoded.  
10100101 -> DEED

**Pseudocode**

Huffman(W, n)

Input: A list W of n weights.

Output: An extended binary tree T with weights

taken from W that gives the minimum weighted path length.

**Procedure:**

Create list F from singleton trees formed from elements of W

WHILE (F has more than one element) DO

Find W in F that have minimum values associated with their roots

Construct new tree T by creating a new node and setting T1 and T2 as its children

Let the sum of the values associated with the roots of T1 and T2 be associated with the root of T

Add T to F

OD

Huffman := tree stored in F

Example:

[**http://homes.sice.indiana.edu/yye/lab/teaching/spring2014-C343/huffman.php**](http://homes.sice.indiana.edu/yye/lab/teaching/spring2014-C343/huffman.php)

1. **Implementation of Activity selection problem**

**Problem: Given n activities with their start and finish times. Select the maximum number of activities that can be performed by a single person, assuming that a person can only work on a single activity at a time.**

**Algorithm**

* 1. Sort the activities according to their finishing time
  2. Select the first activity from the sorted array and print it.
  3. Do following for remaining activities in the sorted array.

If the start time of this activity is greater than or equal to the finish time of previously selected activity then select this activity and print it.

**Example 1 :** Consider the following 3 activities sorted by

by finish time.

start[] = {10, 12, 20};

finish[] = {20, 25, 30};

A person can perform at most **two** activities. The

maximum set of activities that can be executed

is {0, 2} [ These are indexes in start[] and

finish[] ]

**Example 2 :** Consider the following 6 activities

sorted by by finish time.

start[] = {1, 3, 0, 5, 8, 5};

finish[] = {2, 4, 6, 7, 9, 9};

A person can perform at most **four** activities. The

maximum set of activities that can be executed

is {0, 1, 3, 4} [ These are indexes in start[] and

finish[] ]

**Analyze time complexity of the problem.**

1. **Implementation of** [**Job Sequencing Problem**](http://www.geeksforgeeks.org/job-sequencing-problem-set-1-greedy-algorithm/)

**Problem**: **Given an array of jobs where every job has a deadline and associated profit if the job is completed before the deadline. It is also given that every job takes single unit of time, so the minimum possible deadline for any job is 1. How to maximize total profit if only one job can be scheduled at a time.**

**Algorithm**

1) Sort all jobs in decreasing order of profit.

2) Initialize the result sequence as first job in sorted jobs.

3) Do following for remaining n-1 jobs

If the current job can fit in the current result sequence without missing the deadline, add current job to the result.

Else ignore the current job.

**Example:**

Input: Five Jobs with following deadlines and profits

JobID Deadline Profit

a 2 100

b 1 19

c 2 27

d 1 25

e 3 15

Output: Following is maximum profit sequence of jobs

c, a, e

1. **Implementation of** [**Greedy Algorithm to find Minimum number of Coins**](http://geeksquiz.com/greedy-algorithm-to-find-minimum-number-of-coins/)

**Problem**: Given a value V, if we want to make change for V Rs, and we have infinite supply of each of the denominations in Indian currency, i.e., we have infinite supply of { 1, 2, 5, 10, 20, 50, 100, 500, 1000} valued coins/notes, what is the minimum number of coins and/or notes needed to make the change?

**Examples:**

Input: V = 70

Output: 2

We need a 50 Rs note and a 20 Rs note.

Input: V = 121

Output: 3

We need a 100 Rs note, a 20 Rs note and a

1 Rs coin.

Start from largest possible denomination and keep adding denominations while remaining value is greater than 0.

**Algorithm:**

1) Initialize result as empty.

2) find the largest denomination that is

smaller than V.

3) Add found denomination to result. Subtract

value of found denomination from V.

4) If V becomes 0, then print result.

Else repeat steps 2 and 3 for new value of V

1. **Implementation of** [**Minimum Number of Platforms Required for a Railway/Bus Station**](http://www.geeksforgeeks.org/minimum-number-platforms-required-railwaybus-station/)

**Problem**: Given arrival and departure times of all trains that reach a railway station, find the minimum number of platforms required for the railway station so that no train waits.

arr[] = {9:00, 9:40, 9:50, 11:00, 15:00, 18:00}

dep[] = {9:10, 12:00, 11:20, 11:30, 19:00, 20:00}

**All events sorted by time.**

Total platforms at any time can be obtained by subtracting total departures from total arrivals by that time.

Time Event Type Total Platforms Needed at this Time

9:00 Arrival 1

9:10 Departure 0

9:40 Arrival 1

9:50 Arrival 2

11:00 Arrival 3

11:20 Departure 2

11:30 Departure 1

12:00 Departure 0

15:00 Arrival 1

18:00 Arrival 2

19:00 Departure 1

20:00 Departure 0

Minimum Platforms needed on railway station = Maximum platforms needed at any time = 3

Time Complexity: O(nLogn), assuming that a O(nLogn) sorting algorithm for sorting arr[] and dep[].

1. **Implementation of** [**Graph coloring**](http://www.geeksforgeeks.org/graph-coloring-set-2-greedy-algorithm/)

**Unfortunately, there is no efficient algorithm available for coloring a graph with minimum number of colors as the problem is a known**[**NP Complete problem**](http://www.geeksforgeeks.org/np-completeness-set-1/)**. Greedy Algorithm to assign colors doesn’t guarantee to use minimum colors, but it guarantees an upper bound on the number of colors. The basic algorithm never uses more than d+1 colors where d is the maximum degree of a vertex in the given graph.**

**Algorithm:**

1. Color first vertex with first color.

2. Do following for remaining V-1 vertices.

a) Consider the currently picked vertex and color it with the

lowest numbered color that has not been used on any previously

colored vertices adjacent to it. If all previously used colors

appear on vertices adjacent to v, assign a new color to it.

Time Complexity: O(V^2 + E) in worst case.

1. **Implementation of** [**Fractional Knapsack Problem**](http://www.geeksforgeeks.org/fractional-knapsack-problem/)

**Problem: Given weights and values of n items, we need to put these items in a knapsack of capacity W to get the maximum total value in the knapsack.**

**In the**[**0-1 Knapsack problem**](http://www.geeksforgeeks.org/dynamic-programming-set-10-0-1-knapsack-problem/)**, we are not allowed to break items. We either take the whole item or don’t take it.**

Input:

Items as (value, weight) pairs

arr[] = {{60, 10}, {100, 20}, {120, 30}}

Knapsack Capacity, W = 50;

Output:

Maximum possible value = 220

by taking items of weight 20 and 30 kg for 0-1 Knapsack

Output :

Maximum possible value = 240

By taking full items of 10 kg, 20 kg and

2/3rd of last item of 30 kg

Calculate the ratio value/weight for each item and sort the item on basis of this ratio. Then take the item with the highest ratio and add them until we can’t add the next item as a whole and at the end add the next item as much as we can. Which will always be the optimal solution to this problem. As main time taking step is sorting, whole problem can be solved in O(n log n) only.

1. **Implementation of** [**Minimize Cash Flow among a given set of friends who have borrowed money from each other**](http://www.geeksforgeeks.org/minimize-cash-flow-among-given-set-friends-borrowed-money/)

**Problem**: Given a number of friends who have to give or take some amount of money from one another. Design an algorithm by which the total cash flow among all the friends is minimized.

**Algorithm**

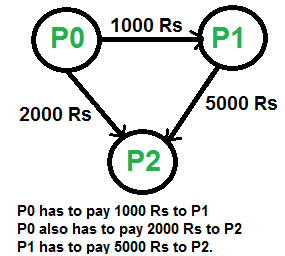
Do following for every person Pi where i is from 0 to n-1 for the below given example.  
1) Compute the net amount for every person. The net amount for person ‘i’ can be computed be subtracting sum of all debts from sum of all credits.

**2)**Find the two persons that are maximum creditor and maximum debtor. Let the maximum amount to be credited maximum creditor be maxCredit and maximum amount to be debited from maximum debtor be maxDebit. Let the maximum debtor be Pd and maximum creditor be Pc.

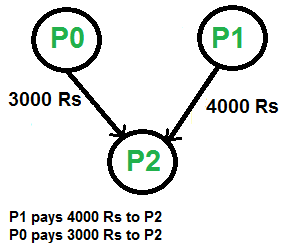
**3)**Find the minimum of maxDebit and maxCredit. Let minimum of two be x. Debit ‘x’ from Pd and credit this amount to Pc

**4)**If x is equal to maxCredit, then remove Pc from set of persons and recur for remaining (n-1) persons.  
**5)**If x is equal to maxDebit, then remove Pd from set of persons and recur for remaining (n-1) persons.

**Example**:  
Following diagram shows input debts to be settled.

[](http://www.geeksforgeeks.org/wp-content/uploads/cashFlow.png)

Above debts can be settled in following optimized way

[](http://www.geeksforgeeks.org/wp-content/uploads/cashFlow1.png)

**Time Complexity:** O(N2) where N is the number of persons.

1. **Implementation of** [**Find minimum time to finish all jobs with given constraints**](http://www.geeksforgeeks.org/find-minimum-time-to-finish-all-jobs-with-given-constraints/)

**Problem**: Given an array of jobs with different time requirements. There are K identical assignees available and we are also given how much time an assignee takes to do one unit of job. Find the minimum time to finish all jobs with following constraints.

* An assignee can be assigned only contiguous jobs. For example, an assignee cannot be assigned jobs 1 and 3.
* Two assignees cannot share (or co-assigned) a job, i.e., a job cannot be partially assigned to one assignee and partially to other.

**Input :**

K: Number of assignees available.

T: Time taken by an assignee to finish one unit of job

job[]: An array that represents time requirements of

different jobs.

**Example**:

Input: k = 2, T = 5, job[] = {4, 5, 10}

Output: 50

The minimum time required to finish all jobs is 50. There are 2 assignees available. We get this time by assigning {4, 5} to first assignee and {10} to second assignee.

Input: k = 4, T = 5, job[] = {10, 7, 8, 12, 6, 8}

Output: 75

We get this time by assigning {10} {7, 8} {12} and {6, 8}